PMT

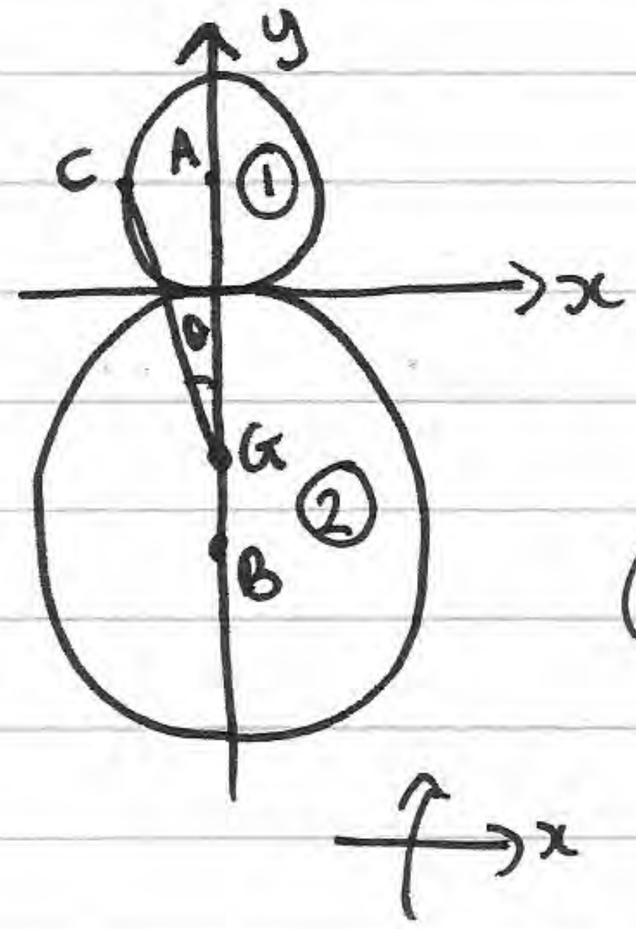
M2 JUNE 01

1) $r = (t^2+2t)i + (t-2t^2)j$

$v = \frac{dr}{dt} = (2t+2)i + (1-4t)j$

$a = \frac{dv}{dt} = 2i - 4j \Rightarrow |a| = \sqrt{2^2+4^2} = \underline{2\sqrt{5}\ ms^{-2}}$

2)



u = mass per unit cm^2

① $M = 100\pi u$ $g_1(0,10)$

② $M = 400\pi u$ $g_2(0,-20)$

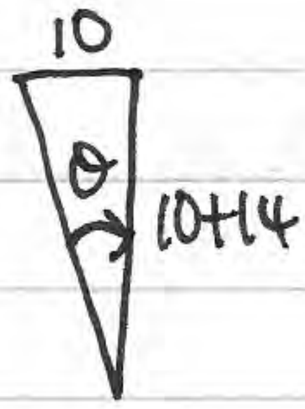
①+② $M = 500\pi u$ $G(0,\bar{y})$

$100\pi u g \times 10 + 400\pi u g \times -20 = 500\pi u g \bar{y}$

$\Rightarrow 1000 - 8000 = 500\bar{y} \Rightarrow \bar{y} = -14$

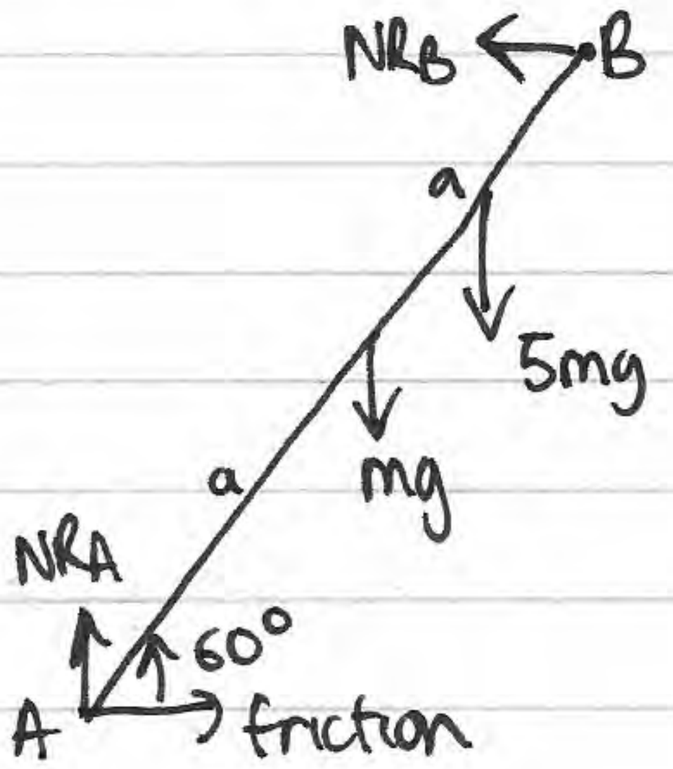
$\Rightarrow$ 6cm above B

b)



$\theta = \tan^{-1}\left(\frac{10}{24}\right) = \underline{22.6°}$

3)



$M = \frac{1}{2}$

at greatest value of u, ladder will be in limiting equilibrium

$\Rightarrow$ friction $= f_{max} = \mu N_{RA} = \frac{1}{2} N_{RA}$

$R_f \uparrow = \downarrow \Rightarrow N_{RA} = 6mg \Rightarrow f_{max} = 3mg.$

$\overrightarrow{R_f} = 0 \Rightarrow N_{RB} = f_{max} = 3mg.$

A2 $mg \times a\cos 60 + 5mg \times ua\cos 60 = 3mg \times 2a\sin 60$

$\frac{1}{2}\alpha + \frac{5}{2}u\alpha = 3\sqrt{3}\alpha$ $\quad u = \frac{6\sqrt{3}-1}{5} = \underline{1.88\ (3sf)}$

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4) Mom before + Impulse = Mom after

$\Rightarrow 0.1u + 3.5i + 3j = 0.1(10i + 25j)$

$\Rightarrow 0.1u = -2.5i - 2.5j$ $\Rightarrow u = -25i + 25j$ ms^{-1}

b) $u\uparrow = 25$ $v^2 = u^2 + 2as$

$v\uparrow = 0$ $0 = 25^2 - 19.6s$ $s \Rightarrow 31.9$m

$a\uparrow = -9.8$ $\Rightarrow$ 32.9m above ground.

c) $s\uparrow = 0$ $s = ut + \frac{1}{2}at^2$ $\Rightarrow 0 = 25t - 4.9t^2$

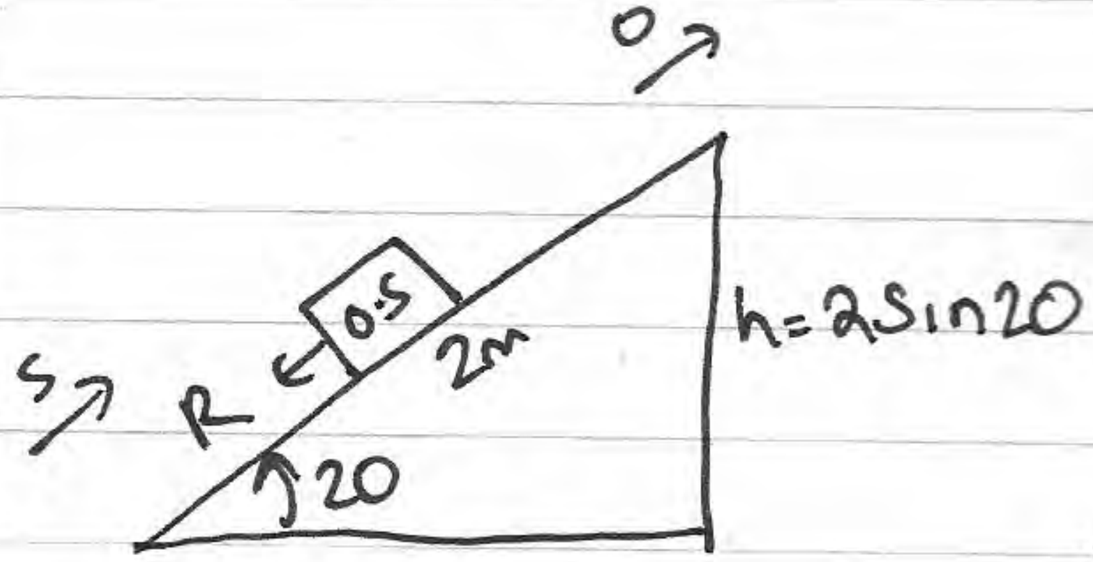
$0 = t(25 - 4.9t)$

$\Rightarrow t = 5.10\ldots$

$\vec{H}$ Vel $= 10$ $t = 5.10..$

$x = 10 \times 5.10..$ $x = 51$m (2sf)

5)



KE_A - Wd against R = PEB

$\frac{1}{2}(0.5)S^2 - R \times 2 = 0.5g(2\sin 20)$

$2R = \frac{25}{4} - g\sin 20$ $\Rightarrow R = 1.45$N (3sf)

b) $\frac{1}{2}(0.5)S^2 - R \times S = 0.5g(S\sin 40)$

$\frac{1}{2}(0.5)S^2 = (0.5g\sin 40 + R)S$ $\Rightarrow S = 1.36$m (3sf)

6) A (2m) $\rightarrow 2u$ $\rightarrow v_A$ B (4m) $\rightarrow u$ $\rightarrow v_B$

$e = \frac{1}{2} = \frac{v_B - v_A}{u}$ $\Rightarrow u = 2v_B - 2v_A$

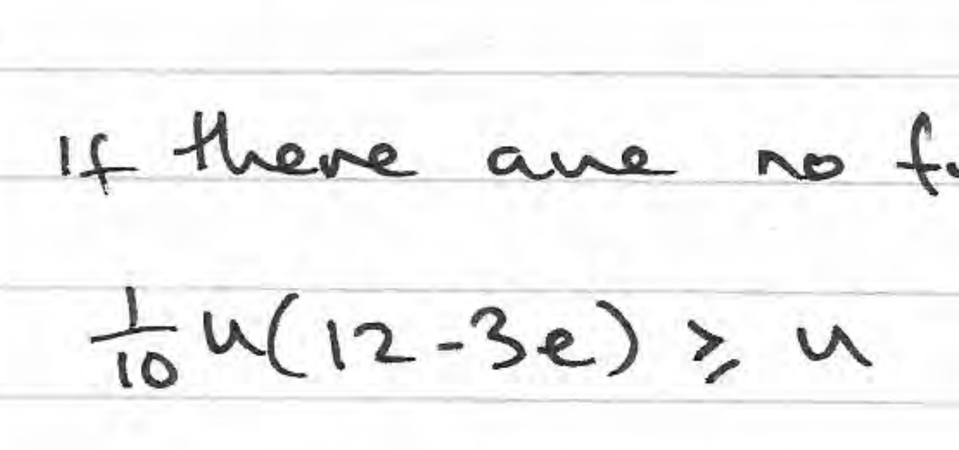
$2v_B = u + 2v_A$

CLM $\Rightarrow 4mu + 4mu = 2mv_A + 4mv_B$

$\Rightarrow 8mu = 2mV_A + 2mu + 4mV_A \Rightarrow 8\cancel{m}u = 6\cancel{m}V_A + 2\cancel{m}u$

$\Rightarrow 6u = 6V_A \Rightarrow V_A = u$

$\Rightarrow 2V_B = 3u \Rightarrow V_B = \frac{3}{2}u$ #



$e = \frac{V_C - V_B}{\frac{3}{2}u} \Rightarrow 3eu = 2V_C - 2V_B$

CLM $\Rightarrow 6\cancel{m}u = 4\cancel{m}V_B + \cancel{m}V_C \Rightarrow V_C = 6u - 4V_B$

$\Rightarrow 3eu = 12u - 8V_B - 2V_B \Rightarrow 10V_B = 12u - 3eu$

$\Rightarrow V_B = \frac{1}{10}u(12 - 3e)$

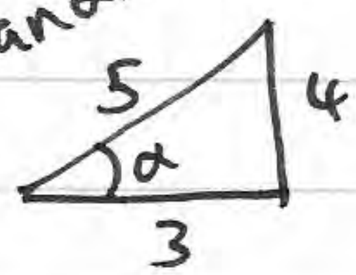
If there are no further collisions $V_B \geq u$

$\frac{1}{10}u(12-3e) \geq u \Rightarrow 12 - 3e \geq 10 \Rightarrow 2 \geq 3e$

$\Rightarrow e \leq \frac{2}{3}$

$\underline{0 < e \leq \frac{2}{3}}$

7) $\tan\alpha = \frac{3}{4}$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

(V↑)

$u\uparrow = 23.75 \times \frac{4}{5}$

$s\uparrow = -2.4$

$a\uparrow = -9.8$

$S = ut + \frac{1}{2}at^2 \Rightarrow -2.4 = 19t - 4.9t^2 \Rightarrow 4.9t^2 - 19t - 2.4 = 0$

$t = \frac{19 + \sqrt{19^2 - 4(4.9)(-2.4)}}{9.8} \rightarrow t = \underline{4\ sec}$

b) $V = \int a\,dt = -\frac{1}{12}t^3 + C \quad V = 18, t = 0 \Rightarrow C = 18$

$V = -\frac{1}{12}t^3 + 18$

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when $t=T$ $V=0$ $\quad 0 = -\frac{1}{12}T^3 + 18$

$\Rightarrow T^3 = 216 \Rightarrow \underline{T = 6\text{sec}}$

d) $\vec{M}$ $\quad \vec{u} = 23.75 \times \frac{3}{5} = 14.25$

$t=4$

$x = 14.25 \times 4 = 57\text{m}$ $\quad AC = 57\text{m}.$

$V = -\frac{1}{12}t^3 + 18 \quad S = \int v\,dt = -\frac{1}{48}t^4 + 18t + C$

$S=0, t=0 \Rightarrow C=0 \quad \Rightarrow S = -\frac{1}{48}t^4 + 18t$

$t=4 \quad S = 66\frac{2}{3}\text{m}.$

Lorry is $66\frac{2}{3}$m from A when C hits the ground

$\therefore$ Lorry is $66\frac{2}{3} - 57 = \underline{9\frac{2}{3}\text{m ahead of C}}$